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Small-x behaviour of the polarized photon structure function $F_3^\gamma(x, Q^2)$

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Abstract. We study the small-x behaviour of the polarized photon structure function F_3^γ, measuring the gluon transversity distribution, in the leading logarithmic approximation of perturbative QCD. There are two contributions, both arising from two-gluon exchange. The leading contribution to small-x is related to the BFKL pomeron and behaves like $x^{-1-\omega_2}$, $\omega_2 = \mathcal{O}(\alpha_S)$. The other contribution includes in particular the ones summed by the DGLAP equation and behaves like $x^{1-\omega_0^{(+)}}$, $\omega_0^{(+)} = \mathcal{O}(\sqrt{\alpha_S})$.

1 Introduction

The deep-inelastic scattering off a photon attracted much attention over 20 years. The idea of extracting the direct contribution to the structure function, which is calculable from theory without additional assumptions [1], motivated experimental studies more involved compared to the standard lepton-nucleon reactions ([2] and references therein).

The extension of the deep-inelastic photon cross section on a target of spin 1 involves structure functions which have no analogon in the usual spin $\frac{1}{2}$ case. The Lorentz structure of photon-photon amplitudes have been studied by many authors, e.g. [3]. The expression for the deep-inelastic cross section on general spin 1 target is given in [4]. We recall the expression for the photon case

$$\frac{d^2\,\sigma}{dq^2\,dx} \sim L^{\mu\nu}\,W_{\mu\nu}\ ,$$
$$W^{\mu\nu} = \left(F_1 - \frac{1}{2}F_3^\gamma\right)\left(g^{\mu\nu} - \frac{q^\mu q^\mu}{q^2}\right)\epsilon_\perp^{*\rho}\epsilon_{\perp\rho}$$
$$-\left(F_2 - \frac{1}{2}F_3^\gamma\right)\left(p^\mu - \frac{pq}{q^2}q^\mu\right)\left(p^\nu - \frac{pq}{q^2}q^\nu\right)\epsilon_\perp^{*\rho}\epsilon_{\perp\rho}$$
$$+\frac{1}{2}F_3^\gamma\left(\epsilon_\perp^{*\mu}\epsilon_\perp^\nu + \epsilon_\perp^{*\nu}\epsilon_\perp^\mu\right) + i\,g_1\,\epsilon^{\mu\nu\lambda\alpha}q_\lambda\,S_\alpha\ . \qquad (1.1)$$

Here $\epsilon_\perp^\mu$ is the transverse (with respect to p and q) part of the target photon helicity vector. S_α is the spin vector given by $S_\alpha = i\,\epsilon^{\alpha\beta\gamma\tau}\epsilon_{\perp\beta}\epsilon^*_{\perp\gamma}p_\tau$.

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Comparing with the proton target case we see that indeed F_3^γ is the interesting new structure function. It can be measured if the target photon is polarized; the polarization of the virtual photon, i.e. of the lepton scattered with large momentum transfer, is not necessary. This structure function measures the parton polarization in an essentially different way compared to g_1. It is related to the virtual photon - photon scattering amplitude with helicity flip [4, 5]. In order to give this amplitude the probability interpretation of a parton distribution one has to change the basis from helicity (circular polarization) to linear polarizations. F_3^γ measures the asymmetry of linear polarizations, the transversity of gluons.

F_3^γ is similar to the structure function h_1, which appears in the Drell-Yan cross-section and measures the transversity of quarks [6]. We show that there is a far-reaching analogy of the small-x behaviour of both F_3^γ and h_1. The small-x behaviour of h_1 has been analyzed recently [7]. There are recent results on the Q^2 evolution of h_1 [8]. Related questions have been discussed in the workshop on spin physics at HERA [9].

In the present paper we investigate the polarized photon structure function F_3^γ at small-x. In this asymptotics it is appropriate to consider the t-channel exchange of the corresponding forward scattering amplitude and to study the leading singularities on the complex angular momentum plane of the corresponding t-channel partial wave.

In general, P parity and a minimal angular momentum are the relevant t-channel quantum numbers for polarized structure functions. The unpolarized parton distributions are related to positive parity exchange, the helicity distributions to negative parity exchange. The second type of polarized parton distributions, the transverse polarization asymmetries, are related to the exchange of longitudinal angular momentum 1 (quark transversity) or 2 (gluon transversity).

It is convenient to study the perturbative Regge asymptotics in the framework of the high-energy effective action [10,11], leading in particular to reggeized gluons and quarks. Two leading reggeized gluons interacting via effective vertices result in the BFKL pomeron. Non-leading gluonic reggeons have been studied recently [12]. The latter are relevant in particular for polarized structure functions since they are able to to transfer odd parity or non-vanishing longitudinal angular momentum projection (s-channel helicity) σ. The leading gluonic reggeon corresponds to $\sigma = 0$ and the leading reggeized quark to $\sigma = \frac{1}{2}$. The multiple exchange of reggeons gives rise to the tree-level energy behaviour s^{α_0}, where

$$\alpha_0 = 1 - \sum \sigma_i \quad . \tag{1.2}$$

This can be considered as the extension of the known Asimov rule [13] to non-leading exchanges. Helicity 2 exchange by two gluons, relevant for our case, can occur in two ways:

(1) by non-leading reggeons ($\sigma_1 = 0, \sigma_2 = 2$), $\alpha_0 = -1$

(2) by leading reggeons ($\sigma_1 = \sigma_2 = 0$) with a non-vanishing longitudinal projection of orbital momentum, $\alpha_0 = +1$.

We shall encounter contributions of both cases.

After recalling in Sect. 2 the known results about the photon spin-flip amplitude [3,14,4,5], about the one-loop coefficient function and the parton splitting function [16, 15,17], we sum in Sect. 3 the double-logarithmic corrections. This concerns a contribution to F_3^γ behaving like $x^{1-\omega_0^{(+)}}$, corresponding to the case (1) above with $\alpha_0 = -1$. We calculate the partial wave whose singularity position gives rise to this behaviour and to the related resummed anomalous dimensions near angular momentum $j = -1$.

The contribution of the BFKL pomeron to the photon spin-flip amplitude is known [22]. In Sect. 4 we recall the impact factors relevant for the photon structure function and the branch point singularity of the conformal spin or helicity $n = 2$ term in the BFKL solution. This contribution to F_3^γ behaves like $x^{-1-\omega_2}$ corresponding to the case (2) above with $\alpha_0 = +1$.

2 Lowest order contribution and Q^2 evolution

The spin structure function of the photon F_3^γ is given by the imaginary part of the virtual photon-photon forward scattering amplitude with spin flip of both scattered photons [5]

$$W^{\mu\nu;\alpha\beta}\Big|_{\Delta=2} = \left(P^{\mu\nu;\alpha\beta}_{(+-;-+)} + P^{\mu\nu;\alpha\beta}_{(-+;+-)}\right) F_3^\gamma(x, Q^2) \tag{2.1}$$

$$P^{\mu\nu;\alpha\beta}_{(+-;-+)} + P^{\mu\nu;\alpha\beta}_{(-+;+-)} = \frac{1}{2}\left(g_\perp^{\mu\alpha} g_\perp^{\nu\beta} + g_\perp^{\mu\beta} g_\perp^{\nu\alpha} - g_\perp^{\mu\nu} g_\perp^{\alpha\beta}\right) \ .$$

$W^{\mu\nu}$ in (1.1) is related to (2.1) by $W^{\mu\nu} = W^{\mu\nu;\alpha\beta}\epsilon_{\perp\alpha}\epsilon^*_{\perp\beta}$. The transverse subspace is the one orthogonal to the momentum q ($q^2 = -Q^2$) of the virtual photon and p ($p^2 = 0$) of the photon target.

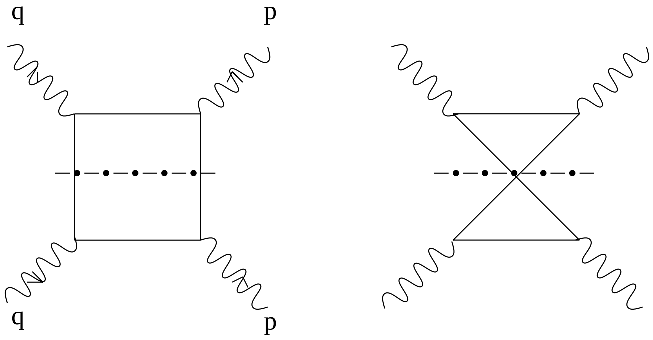


Fig. 1. Fermion loop contribution to the imaginary part of the $\gamma^*\gamma$ forward scattering amplitude

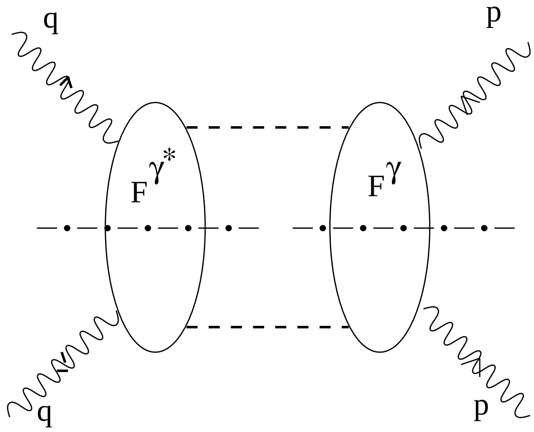


Fig. 2. Two-gluon exchange contribution to $\gamma^*\gamma$ forward scattering

Consider first the lowest order contribution to this amplitude arising from a fermion loop, Fig. 1. This has been calculated by many authors long ago [3,14]. We use calculations by Zima [14] and pick up the projection Eq. (2.1) from the result

$$F_3^{(0)}(x, Q^2) = \frac{e^4}{4\pi^2}\, x^2\, N \ , \tag{2.2}$$

where e is the electromagnetic coupling constant and N denotes the number of colours.

The lowest order fermion loop does not contribute to the anomalous dimension. Notice that the resulting amplitude (2.1), (2.2) is even in exchanging s and u channels. We see that the positive signature amplitude is relevant for the photon spin structure function.

It is known that contributions to the anomalous dimensions (Q^2 evolution) arise from the exchange of two gluons and these contributions are proportional to x, i.e. they dominate (2.2) at small x [15,16].

We calculate the lowest order contribution with two gluon exchange in the asymptotics of small x (Fig. 2).

As building blocks we have the imaginary parts of the fermion loop photon-gluon scattering amplitude, on one side with the virtual photon q and on the other with the real photon p. Both can be read off from [14], where the same projector (2.1) applies ($k_\mu = -\alpha q'_\mu + \beta p_\mu + \kappa_{\perp\mu}$, $q'_\mu = q_\mu - \frac{q^2}{2pq} p_\mu$)

$$F^{\gamma^*}(\beta\, s, \kappa_\perp; Q^2) = 8\, e^2 g^2 \, \frac{\kappa_\perp^2 Q^2}{(s\beta)^2} \ln \frac{s\beta}{\sqrt{\kappa_\perp^2 Q^2}} \,, \qquad Q^2 \gg |\kappa_\perp^2| \,,$$

$$F^\gamma(\alpha\, s, \kappa_\perp; Q_0^2) = -2\, e^2 g^2 \frac{(\kappa_\perp^2)^2}{(s\alpha)^2} \,, \quad Q_0^2 \ll |\kappa_\perp^2| \,. \quad (2.3)$$

F^γ for $Q_0^2 \ll |\kappa_\perp^2|$ takes into account the direct photon contribution. A resolved (non-perturbative) contribution is to be added. The results are approximate according to the Regge kinematics appropriate at small x:

$$s\alpha\beta \ll \kappa_\perp^2 \,, \quad s \gg \kappa_\perp^2 \,, \qquad s\alpha \gg \kappa_\perp^2 \,, \quad s\beta \gg \kappa_\perp^2 + Q^2 \,. \quad (2.4)$$

We obtain ($x = \frac{Q^2}{s}$)

$$\int \frac{d^4 k}{(k^2 + i\epsilon)^2} F^{\gamma^*}(\beta s, \kappa_\perp; Q^2) F^\gamma(\alpha s, \kappa_\perp; Q_0^2) = -e^4 g^4 N^2 \left[2\pi x \ln^2 x + 8\pi x \left(\frac{\pi^2}{12} - 1 \right) + \mathcal{O}(x^2) \right] \quad (2.5)$$

The resulting contribution to the structure function behaves indeed like x with logarithmic corrections.

The Q^2 evolution arises from the interaction of the exchanged gluons with strong ordering of the transverse momenta [18] (see Figs. 3a,b)

$$F_3^\gamma = F_3^{(0)} \otimes F_3^g \,, \quad (2.6)$$

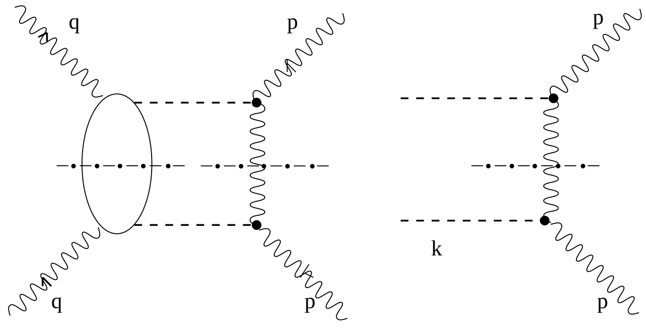
$$\frac{d}{d \ln Q^2} \, F_3^g(x, Q^2) = \frac{g^2(Q^2)}{8\pi^2} \int_x^1 \frac{dz}{z} \, P(\frac{z}{x}) \, F_3^g(z, Q^2) \,.$$

The photon structure function is obtained from the gluon transversity F_3^g by convolution with the coefficient function proportional to $F_3^{(0)}$ (2.2).

The initial-value x-distribution at Q_0^2 for (2.6) is the sum of the perturbative contribution of F^γ (2.3) and a non-perturbative resolved contribution. The evolution kernel $P(z)$ is readily obtained, e.g. by calculating the graph in Fig. 3b in the axial gauge, $A^\mu q'_\mu = 0$,

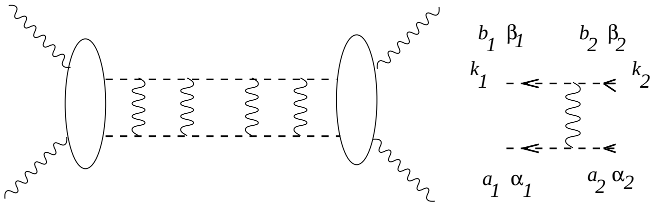
$$P(z) = \frac{2Nz}{1-z} \,. \quad (2.7)$$

The anomalous dimensions corresponding to this result has been presented first in [17] without relation to F_3^γ. Later this splitting function was derived in [15] and in [16]. In (2.7) we do not write the contributions $\sim \delta(z-1)$ which are irrelevant for the small-x asymptotics.



(a) (b)

Fig. 3a,b. One-loop contribution to the Q^2 evolution



(a) (b)

Fig. 4. The gluon ladder contribution

We shall see that this leading $\ln Q^2$ evolution leads to a small-x behaviour of F_3^γ proportional to x up to logarithmic corrections. Correspondingly, the one-loop anomalous dimension has a pole at $j = -1$. The logarithmic correction to the small-x asymptotics are not completely accounted for by (2.7) as we shall discuss below.

3 Double log contributions at small x

3.1 The gluon ladder graphs

We consider the contribution of a s-channel intermediate state of gluons in multi-Regge kinematics to the interaction of the exchanged gluons (see Fig. 4a).

From the DGLAP equation (2.6) we see that there is a double log contribution

$$\sim g^2 (\ln \frac{1}{x}) \ln Q^2$$

from each loop arising from the configuration of strongly ordered transverse momenta. We show that there are further double logarithms resulting in a contribution $\sim g^2 \ln^2 \frac{1}{x}$ from each loop which determine the small-x behaviour.

As the first step we analyze the one-rung contribution (Fig. 4b) in the appropriate projection (2.1)

$$\Gamma^{\alpha_2 \alpha_1 \sigma}(k_2, k_1) f^{a_2 a_1 c} \, g^\perp_{\sigma\sigma'} \Gamma^{\sigma' \beta_1 \beta_2}(k_1, k_2) f^{c b_1 b_2} \frac{\delta_{a_1 b_1} \delta_{a_2 b_2}}{N^2 - 1} (P_{(+-;-+)} + P_{(-+;+-)})_{\alpha_1 \beta_1, \alpha_2 \beta_2} = 4N(\kappa_{1\perp}^2 + \kappa_{2\perp}^2) \,, \quad (3.1)$$

where Γ is the triple-gluon vertex. Note that only the transverse gluon polarizations contribute to the s-channel intermediate states.

In order to have a double log contribution from each loop we have to obtain a logarithmic contribution from the transverse momentum $\kappa_{i\perp}$ integral. For this a factor of $\kappa_{i\perp}^2$ from the numerator as is provided by the result (3.1) is essential. The coefficient $4N$ in (3.1) determines the size of the double log contribution.

The ladder graphs shown in Fig. 4a are summed by an integral equation the kernel of which is obtained in an obvious way using (3.1). The peculiarity are the limits of integration in the longitudinal momentum fraction β and the transverse momentum κ, which show whether there are double logs beyond the strong ordering region in κ:

$$x \ll \dots \beta_i \ll \beta_{i+1} \dots \ll 1 \; , \qquad \frac{Q^2}{x} \gg \dots \frac{|\kappa_i^2|}{\beta_i} \gg \frac{|\kappa_{i+1}^2|}{\beta_{i+1}} \dots \gg \mu^2 \; . \tag{3.2}$$

Here and in the following we denote the transverse vectors by means of the complex number ($\kappa = \kappa_\perp^1 + i\kappa_\perp^2$). Indeed we find that there is strong ordering in β's and in $\frac{|\kappa^2|}{\beta}$, which leads to a double log region larger than the one in the DGLAP equation.

The discussion above relies on the simple ladder graphs assuming that there are no further double log contributions. This is indeed the true for the negative signature channel, but not obvious. However we finally have to calculate the positive signature partial wave, as we have pointed out in Sect. 2. We apply the method of [19].

3.2 The soft t-channel intermediate state

The leading $\ln s$ approximation in the considered channel results in a sum of ladder graphs summed by an BFKL-like equation. The ladders are built of effective vertices and reggeized gluons and each of them is a gauge invariant sum (in the considered approximation) of Feynman graphs. The leading $\ln s$ contributions in the vicinity of $j = -1$ and for negative signature, i.e. the $\ln s$ corrections to the power s^{-1} contribution to the amplitude, include in particular the double log contributions in question. Consider the transverse momentum κ_i integral in one of the ladder loop. The region where κ_i is smaller than all other transverse momenta in the loops is to be investigated in particular. The question about the double log beyond the DGLAP equation reduces to the question of whether the transverse momentum integral is logarithmic in this region of smallest κ.

The ladder loop at $\kappa \to 0$, Fig. 5a, is related to the product of graphs with scattering gluons Fig. 5b. In this way the exchanged (t-channel) reggeized gluons are related to scattered (s-channel) gluons. Due to the particular projection (2.1) the gluon scattering is accompanied by helicity flip.

The scattering graph Fig. 5b stands for gauge invariant sum of graphs involving effective scattering vertices.

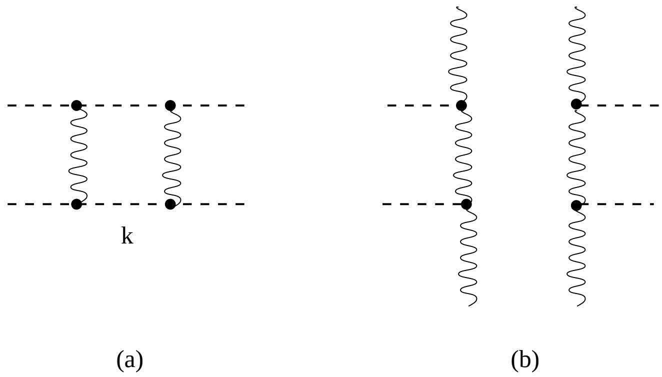


Fig. 5a,b. Gluon ladder element with the smallest transverse momentum **a** and the related gluon scattering graphs **b**

In the tree approximation there are no effective scattering vertices changing helicity with leading (near $j = 1$) or subleading (near $j = 0$) gluonic reggeons [12]. A helicity-flip effective vertex appears only at the next-to-subleading ($j = -1$) level. We conclude that the contribution to the considered channel (2.1) arises from the exchange of one leading ($j \sim 1$) and one next-to-subleading ($j \sim -1$) gluonic reggeon.

Now we consider again the relation of the scattering graphs Fig. 5b to the ladder loop Fig. 5a (t-channel unitarity). In order to get Fig. 5a from Fig. 5b one has to turn the scattering gluons into exchanged reggeons and add the propagators. The trajectory functions $\alpha(\kappa)$ can be disregarded in the double log approximation.

The relation of a leading gluonic reggeon to a scattering gluon in the small κ region has been analysed in [11]. It is important that there arises a factor of κ at each vertex in turning the scattering gluon into the leading gluonic reggeon. The subleading gluonic reggeon is obtained from the scattering gluon without such additional factor. As a result we have a factor $|\kappa|^2$ and together with the two propagators we obtain a logarithmic transverse momentum integral.

We compare the situation to the other channels: In the case of two leading gluonic reggeons we would have instead a factor $|\kappa|^4$ and in the case of two subleading reggeons no additional factors of κ. In both cases no double log contribution from the soft κ region appears.

The scattering graphs Fig. 5b are directly related to the parton splitting kernel $P(z)$ in the DGLAP equation (2.6). Each of the scattering graphs can be compared with the graph Fig. 3b, from which we have obtained $P(z)$ using axial gauge $q'A = 0$. Indeed, the gauge invariant effective scattering vertices coincide with the bare vertices in the corresponding axial gauge.

The double log contribution of the BFKL ladders can be summed by the equation given graphically in Fig. 6.

Introducing the partial waves we can write it in the following form ($\omega = j + 1$)

$$f^{(-)}(\omega) = \frac{g^2 a_0}{\omega} + \frac{1}{8\pi^2} \frac{1}{\omega} \left(f^{(-)}(\omega)\right)^2 \; . \tag{3.3}$$

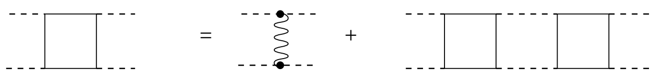


Fig. 6. Equation summing the double log contribution from soft t-channel intermediate states

The Born term corresponds in fact to one of the scattering graphs in Fig. 5b; $\frac{a_0}{j+1}$ is the leading contribution at $j \to -1$ of the moment transformation of $P(x)$

$$\frac{a_0}{j+1} = \int_0^1 d\,x\, x^{j-1}\, P(x)\,|_{j\to -1} \tag{3.4}$$

and we have $a_0 = 2N$. Eq. (3.3) is readily solved

$$f^{(-)}(\omega) = 4\pi^2\omega \left(1 - \sqrt{1 - \frac{4\alpha_S\, N}{\pi\omega^2}}\right) \quad . \tag{3.5}$$

However this is not the final result for the double log contributions to the parton spin-flip amplitude (2.1) because we have calculated so far only the negative signature contribution.

3.3 Soft bremsstrahlung contributions

The leading $\ln s$ effective ladder equation of BFKL type is obtained under the essential assumption of colour singlet state in t-channel. Only in this case the infrared divergencies cancel. To analyze amplitudes and partial waves corresponding to non-singlet channels we restrict the integrations over the transverse momentum κ by the condition $|\kappa|^2 > \mu^2$. In non-singlet channels there are more double log contributions besides of those contained in the ladder. Loops with a single gluon being soft, i.e. carrying the smallest κ, give rise to double logs. This is different from the two-gluon t-channel intermediate state discussed above. The contribution of the soft gluon loop is easily calculated relying on Gribov's bremsstrahlung theorem: The gluon with the smallest transverse momentum is effectively emitted and absorbed from the external lines. With the bremsstrahlung contribution the equation in Fig. 6 generalizes to the one shown in Fig. 7.

The soft bremsstrahlung contributions change the gauge group quantum numbers in the t-channel. Now instead of a single partial wave $f(\omega)$ we have to consider a column vector involving the colour singlet partial wave discussed so far together with the colour octet and further colour channnels

$$\hat{f} = \begin{pmatrix} f \\ f_8 \\ \cdot \\ \cdot \\ \cdot \end{pmatrix} \quad . \tag{3.6}$$

We shall concentrate on these first two entries, disregarding the remaining representations which appear in the symmetric part of the tensor product of two adjoint (gluon) representations.. Let us notice that the octet contribution in question is the one arising by antisymmetrization, i.e. by projection with f^{abc}.

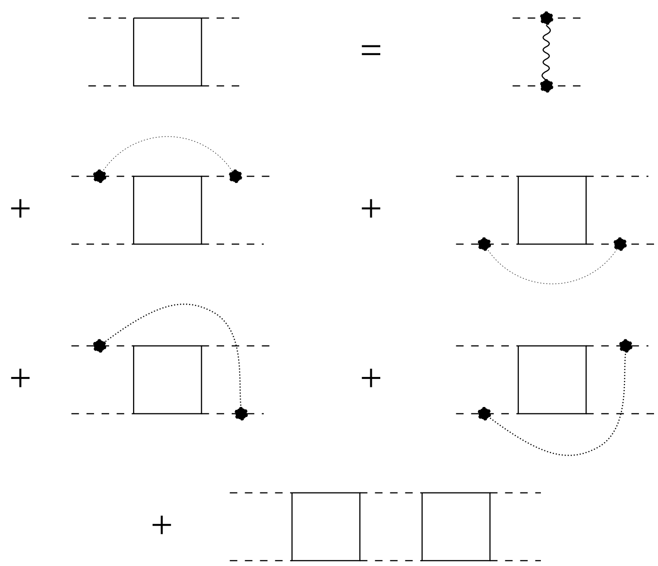


Fig. 7. Equation summing the double log contributions both from soft t-channel intermediate states and from soft bremsstrahlung

In colour space the interaction induced by gluons attached to the external lines is expressed by multiplying the column of partial waves by a matrix. This matrix depends on whether the pair of those external lines belongs to the s- or t- channels. The upper limits of the double logarithmic transverse momentum integrals are respectively s, u or t. Therefore the t-channel type soft gluons are not important in our case. Nevertheless it is reasonable to introduce all three matrices. Obviously $\hat{M}_t$ is diagonal in the representation with definite quantum numbers in the t-channel. Restricting to the two relevant colour channels we have

$$\hat{M}_t = \begin{pmatrix} N & 0 \\ 0 & \frac{N}{2} \end{pmatrix} \quad . \tag{3.7}$$

The sum of the three types of colour interactions taken with the same weight (the interaction in momentum space leads to different weights, however) acts as the identity up to the factor N

$$\hat{M}_s + \hat{M}_u + \hat{M}_t = N\,\hat{I} \tag{3.8}$$

Working out the relations of $SU(N)$ representations leads to

$$\hat{M}_s = \begin{pmatrix} 0 & N \\ \frac{N}{N^2-1} & \frac{N}{4} \end{pmatrix} \quad . \tag{3.9}$$

The complete matrices for all colour representations appearing in the two-gluon exchange are given in [20].

The soft bremsstrahlung contribution is readily calculated in momentum space. The sum of 4 diagrams, two of

s-channel and two of u-channel type, results in

$$\frac{g^2}{4\pi^2}\int_{\mu^2}^{s}\frac{d|\kappa|^2}{|\kappa|^2}\left[\hat{M}_s\ln(\frac{-s}{|\kappa|^2})+\hat{M}_u\ln(\frac{s}{|\kappa|^2})\right]\hat{A}^{(\sigma)}(s,|\kappa|^2)\ . \quad (3.10)$$

The square bracket in (3.10) involves an odd contribution in s, which leads to a change in signature σ. Here $\hat{A}^{(\sigma)}(s,|\kappa|^2)$ is the column vector of (almost on-shell) gluon forward scattering amplitude where the double log corrections are included with κ as the lower cut-off in all transverse momentum integrals

$$\hat{A}^{(\sigma)}(s,|\kappa|^2)=\int_{-i\infty}^{i\infty}\frac{d\,\omega}{2\pi i}\hat{f}^{(\sigma)}(\omega)\left(\frac{s}{|\kappa|^2}\right)^{\omega}\zeta^{(\sigma)}(\omega)\,. \quad (3.11)$$

Taking into account the signature factor approximated for small $\omega=j+1$

$$\zeta^{(-)}(\omega)\approx 1\ ,\qquad \zeta^{(+)}(\omega)\approx -\frac{1}{2}\,i\pi\omega\ , \quad (3.12)$$

we have in terms of partial waves

$$\begin{aligned}\hat{f}^{(-)}(\omega)\Big|_{brems} &= \frac{g^2}{4\pi^2}\left(\hat{M}_s+\hat{M}_u\right)\frac{1}{\omega}\frac{d}{d\omega}\hat{f}^{(-)}(\omega)\\ \hat{f}^{(+)}(\omega)\Big|_{brems} &= \frac{g^2}{4\pi^2}\left(\hat{M}_s+\hat{M}_u\right)\frac{1}{\omega^2}\frac{d}{d\omega}(\omega\hat{f}^{(+)}(\omega))\\ &\quad -\frac{g^2}{4\pi^2}\frac{1}{\omega^2}\left(\hat{M}_s-\hat{M}_u\right)\hat{f}^{(-)}(\omega)\,. \end{aligned} \quad (3.13)$$

The signature changing contribution is relevant in our approximation (small ω) only if multiplied with the negative signature amplitude and therefore leads to an additional inhomogeneous term in the positive signature equation.

We write the resulting partial wave equations in matrix form

$$\begin{aligned}\hat{f}^{(-)}(\omega) &= a\hat{M}_t\frac{g^2}{\omega}+\frac{g^2}{4\pi^2}\left(\hat{M}_s+\hat{M}_u\right)\frac{1}{\omega}\frac{d}{d\omega}\hat{f}^{(-)}(\omega)\\ &\quad +\frac{1}{8\pi^2}\frac{1}{\omega}(\hat{f}^{(-)\,2}(\omega))\\ \hat{f}^{(+)}(\omega) &= a\hat{M}_t\frac{g^2}{\omega}+\frac{g^2}{4\pi^2}\left(\hat{M}_s+\hat{M}_u\right)\frac{1}{\omega^2}\frac{d}{d\omega}(\omega\hat{f}^{(+)}(\omega))\\ &\quad -\frac{g^2}{4\pi^2}\left(\hat{M}_s-\hat{M}_u\right)\frac{1}{\omega^2}\hat{f}^{(-)}(\omega)\\ &\quad +\frac{1}{8\pi^2\omega}(\hat{f}^{(+)\,2}(\omega))\ . \end{aligned} \quad (3.14)$$

$(\hat{f}^{(\pm)\,2})$ denotes the column vector with the squares of the partial waves of the corresponding colour channel. Comparing with a_0 in (3.3) for the colour singlet channel we identify $a=2$.

This is the part of the residue a_0 of the one-loop anomalous dimensions at $j=-1$ not related to colour factors but merely to the helicity state. Comparing to the gluonic contribution to the double log small-x asymptotics of the helicity asymmetry g_1 [21] and to the flavour non-singlet (quark-antiquark) contributions [7] we see that there we have $a=4$ and 1, respectively.

Since the matrix element $(\hat{M}_s+\hat{M}_u)_{11}$ vanishes the equation for the colour singlet component for both signatures is just an algebraic equation of second order. The negative signature solution has been discussed above. For the positive signature we obtain

$$f_0^{(+)}=4\pi^2\omega\left(1-\sqrt{1-\frac{2\alpha_S N}{\pi\omega^2}\left(2-\frac{1}{2\pi^2\omega}f_8^{(-)}(\omega)\right)}\right). \quad (3.15)$$

The solution is expressed in terms of the negative signature octet partial wave. The corresponding equation is differential of Ricatti type. The solution can be expressed in terms of the logarithmic derivative of the parabolic cylinder function $\mathcal{D}_p(z)$ [19]

$$\begin{aligned} f_8^{(-)} &= 4\pi\alpha_S N\frac{d}{d\omega}\ln\left(\exp(\frac{\omega^2}{4\bar{\omega}^2})\mathcal{D}_p(\frac{\omega}{\bar{\omega}})\right)\\ \bar{\omega}^2 &= \frac{\alpha_S N}{2\pi}\ ,\quad p=2\ . \end{aligned} \quad (3.16)$$

The small-x dependence of $F_3^\gamma(x,Q^2)$ is obtained from the partial wave $f_0^{(+)}$ by inverse Mellin transform

$$F_3^\gamma(x,Q^2)\sim\int_{-i\infty}^{i\infty}\frac{d\,\omega}{2\pi i}\,f_0^{(+)}(\omega)\zeta^{(+)}(\omega)\,x^{-\omega}\ . \quad (3.17)$$

In the relation we did not include explicitely the convolution of the reggeon Green function $f_0^{(+)}$ with the appropriate impact factors, which is not important neither for the leading small-x behaviour nor for the leading term of the anomalous dimensions. The small-x asymptotics is determined by the right-most singularity $\omega_0^{(+)}$ of $f_0^{(+)}(\omega)$

$$F_3^\gamma(x,Q^2)\Big|_{double-log}\sim x^{1-\omega_0^{(+)}}\ . \quad (3.18)$$

$\omega_0^{(+)}$ is close to $\omega_0^{(-)}$, the branch point of $f_0^{(-)}(\omega)$

$$\omega_0^{(-)2}=\frac{4\alpha_S N}{\pi}\ . \quad (3.19)$$

Assuming $\alpha_S=0.2$, $N=3$ we have $\omega_0^{(-)}\approx 0.87$. As a rough estimate of the signature changing contribution we replace $f_8^{(-)}$ by its Born term. Unlike the case of two fermion exchange the latter is not suppressed in the large N approximation, therefore this estimate is worse in the present case. We obtain $\omega_0^{(+)}\approx 0.61$.

In the double-log approximation also the resummed anomalous dimensions $\nu^{(+)}(j)$ near $j=-1$ are obtained in terms of the partial wave $f_0^{(+)}(\omega)$, $\omega=j+1$,

$$\nu^{(+)}(-1+\omega)=\frac{1}{8\pi^2}\,f_0^{(+)}(\omega)\ . \quad (3.20)$$

4 The perturbative pomeron contribution

We calculate in the leading $\ln s$ approximation the contribution to the virtual photon spin-flip amplitude (2.1) with the asymptotics $s^{1+\omega_2}$ leading to a contribution to the structure function behaving like $x^{-1-\omega_2}$. There are well known results for the real and virtual photon impact factors with the exchange of two leading gluonic reggeons, describing the coupling of the scattering photon to the BFKL pomeron [24] via a quark loop [25,22,23]. These impact factors involve a helicity-flip contribution at vanishing momentum transfer.

For a virtual photon the helicity-flip ($\Delta\lambda = \pm 2$) contribution to the impact factor reads

$$\Phi_A^{(+,-)}(\kappa, Q^2) = \frac{\alpha_{e.m.}\alpha_S}{\sqrt{2}\,\pi} \cdot 4 \sum_q \int_0^1 dx \int_0^1 dy \times \frac{x(1-x)y(1-y)\,\kappa^{*2}}{|\kappa|^2 x(1-x) + Q^2 y(1-y) + m_q^2}$$
$$\Phi^{(-,+)}(\kappa, Q^2) = \left(\Phi^{(+,-)}(\kappa, Q^2)\right)^* , \quad Q^2 \gg m_\rho^2 \ . \quad (4.1)$$

For quasi-real photons ($Q^2 \leq m_\rho^2$) this expression is applicable only for the heavy flavour contribution. Non-perturbative contributions are essential and can be roughly described by applying the Borel transform with respect to Q^2 to (4.1) and identifying the variable conjugated to Q^2 with the ρ-meson mass m_ρ [22],

$$\Phi_B^{(+,-)}(\kappa, Q^2) = \frac{\alpha_{e.m.}\alpha_S}{\sqrt{2}\,\pi} \cdot 4 \cdot N_q \cdot \int_0^1 dx \int_0^1 dy \times \frac{x(1-x)\kappa^{*2}}{m_\rho^2} \exp\left(-\frac{|\kappa|^2\, x(1-x)}{m_\rho^2\, y(1-y)}\right) + H.Q. \ , \quad Q^2 \leq m_\rho^2 \ . \quad (4.2)$$

Here N_q is the number of light quarks ($N_q = 3$) and $H.Q.$ denotes the expression (4.1) with the sum now restricted to the heavy quark flavours.

In the Regge asymptotics the partial wave of the photon spin-flip forward scattering amplitude can be calculated from the impact representation

$$f_{+,-;-,+}(\omega, Q^2) = \int \frac{d^2\kappa_1\ d^2\kappa_2}{|\kappa_1|^4\ |\kappa_2|^4} \Phi_A^{(+,-)}(\kappa_1, Q^2) \times F(\omega; \kappa_1, \kappa_2)\ \Phi_B^{(-,+)}(\kappa_2, Q_0^2) \ . \quad (4.3)$$

In our case the reggeon Green function $F(\omega; \kappa_1, \kappa_2)$ is obtained from the well known solution of the BFKL equation [23,24]. In the sum over the conformal spin n only the term $n = 2$ contributes

$$F(\omega; \kappa_1, \kappa_2) = \frac{1}{2\pi^2} \int \frac{d\nu\ |\kappa_1^2|^{-\frac{1}{2}+i\nu}\ |\kappa_2^2|^{-\frac{1}{2}-i\nu} \kappa_1^2\ \kappa_2^{*2}}{\omega - \frac{g^2 N}{8\pi^2}\Omega(2,\nu)} ,$$
$$\Omega(n,\nu) = 4\psi(1) - \psi(\frac{1}{2} + i\nu + \frac{n}{2}) - \psi(\frac{1}{2} - i\nu + \frac{n}{2}) - \psi(\frac{1}{2} + i\nu - \frac{n}{2}) - \psi(\frac{1}{2} - i\nu - \frac{n}{2}) \ . \quad (4.4)$$

The spin-flip partial wave has a cut with the right branch point at angular momentum $j = 1 + \omega_2$,

$$\omega_2 = \frac{g^2\, N}{8\pi^2}\Omega(2,\nu) = \frac{g^2\, N}{\pi^2}(\ln 2 - 1) \ . \quad (4.5)$$

Choosing $\alpha_S = \frac{g^2}{4\pi} = 0.2$ as in the usual estimate for the BFKL pomeron intercept and $N = 3$ we have $\omega_2 = -0.23$.

Investigating the Q^2 behaviour of (4.4) we find that this perturbative pomeron contribution does not influence the anomalous dimension (in the vicinity of moment number $j = 1$) at the leading $\ln Q^2$ level. Its influence on the anomalous dimension arises at the next-to-leading level (as a pole term $\sim \frac{1}{j-1}$) from the iteration of both considered contributions.

5 Discussion

There are two contributions to the small-x asymptotics of the polarized photon structure function $F_3^\gamma(x, Q^2)$

$$F_3^\gamma(x, Q^2) = F_3^\gamma(x, Q^2)\Big|_{double-log} + F_3^\gamma(x, Q^2)\Big|_{BFKL} \ . \quad (5.1)$$

The first is closely related to the DGLAP evolution. The small-x asymptotics is obtained in the double logarithmic approximation extending the double-log contribution to the DGLAP equation beyond the region of strong ordering in the transverse momenta. The t-channel partial wave describing this contribution and the related anomalous dimension near angular momentum $j = -1$ in all orders of perturbation theory have been calculated. The small-x behaviour of this contribution is found to be $x^{1-\omega_0^{(+)}}$, where the displacement $\omega_0^{(+)}$ is of order $\sqrt{\alpha_S}$ and is estimated to be $\omega_0^{(+)} \approx 0.6$.

The second contribution is not directly related to the DGLAP evolution. Its trace appears in the anomalous dimensions as pole terms at $j = +1$ starting from the two-loop approximation. This contribution arises from the conformal spin $n = 2$ term of the BFKL pomeron solution [23,24]. It is a term which is not essential in the usual phenomenological applications of the BFKL pomeron. Therefore the experimental study of $F_3^\gamma(x, Q^2)$ would allow to test the detailed structure of the BFKL pomeron. This contribution dominates at small x and behaves like $x^{-1-\omega_2}$, where the displacement ω_2 is of order α_S and is estimated to be $\omega_2 \approx -0.23$.

Both contributions arise from the exchange in t-channel of two (reggeized) gluons interacting by s-channel gluons. s-channel helicity $\sigma = 2$ is transferred in both cases.

In the first case one of the reggeized gluons is the leading one carrying $\sigma = 0$ and the other a twice-subleading one carrying $\sigma = 2$. The latter couples to gluons scattering with helicity flip, whereas the first does not feel the helicity of scattering partons. The resulting Regge singularity is a branch cut at $j = -1 + \omega_0^{(+)}$. In the second case both reggeized gluons are the leading ones ($\sigma = 0$), which are, however, in a state with the longitudinal projection of the orbital angular momentum equal to 2. The resulting Regge singularity is a branch cut at $j = 1 + \omega_2$.

We notice that also in the small-x asymptotics of the structure function $h_1(x, Q^2)$ measuring the quark transversity we have encountered two contributions in analogy to (5.1). There we have two reggeized quarks in the t-channel which carry s-channel helicity $\sigma = 1$. The first contribution ($\sim x$ plus corrections) related to the DGLAP evolution, arises from the exchange of one leading and one subleading quark reggeon. The second contribution (constant in x) arises from two leading quarks with parallel helicities.

We have given the couplings (impact factors) of these exchanges to the scattering photons explicitely. We did not perform here a numerical analysis of the expressions obtained for the amplitude related to $F_3^\gamma(x, Q^2)$. It is clear, however, that the couplings of the angular momentum 2 state are weaker. Therefore we expect that the first contribution ($\sim x^{1-\omega_0^{(+)}}$) will dominate at not too small values of x and will be overcome by the second ($\sim x^{-1-\omega_2}$) only at very small x.

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